Example:

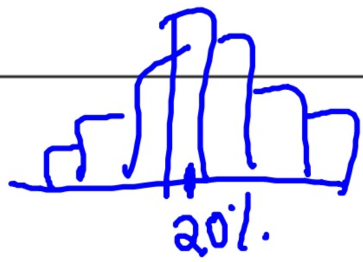
The Mars company claims that there are 20% orange in their M&Ms bags.

I take a sample of 200 M&Ms and get 11 orange M&Ms.

What is $\hat{p}$? ___5.5%___ Do you think the claim is true? Why? NO

What if I take a new sample of 200 M&Ms and get 20 orange M&Ms.

What is $\hat{p}$? ___10%___ Do you think the claim is true? Why? NO



What if I take a new sample of 200 M&Ms and get 30 orange M&Ms.

What is $\hat{p}$? ___15%___ Do you think the claim is true? Why?

Yes

What if I take a new sample of 200 M&Ms and get 36 orange M&Ms.

What is $\hat{p}$? ___18%___ Do you think the claim is true? Why?

Yes

$p = 20\%$

$\hat{p} = \_\_\_\%$

9.2: Hypothesis Tests aka Tests of Significance

With confidence intervals, we don't know....

p, the true percent in the population

$\hat{p} \pm m = (a, b)$

And we are trying to...

estimate the true value (p) by using samples and calculating $\hat{p}$

But what if we have a value of p?

If we have a claim for p, we want to test the claim to see if it is true.

TESTS OF SIGNIFICANCE

Uses/Purposes:

What are these tests used for?

to decide whether a claim is true or not

What do these tests compare?

sample statistic ($\hat{p}$) to a population paramter (p)

$p = 20\%$

$\hat{p} = 21\%$

5%

What are the 4 components of the test?

1) Hypotheses
2) Test Statistic
3) P-Value (probability)
4) Conclusion

COMPONENTS:

1) Hypotheses

What do the hypotheses always describe?

population parameter /claim (p)

What types of symbols do the hypotheses always use?

Greek letters p μ

Null Hypothesis

FORM: Ho: parameter = #

Assumed... to be true, until we can prove it false

For this chapter... Ho: p = %

$H_0: p = 20\%$

0.20

Alternative Hypothesis:

What is it?

What we suspect/think is actually true

$H_0: p = 20\%$
$H_a: p > 20\%$

Symbol: Ha

FORM: Two types of alternative hypotheses:

One sided:	Two sided:
Ha: p > %	Ha: p ≠ %
$p > 20\%$	$p \neq 20\%$
Ha: p < %	
$p < 20\%$	

different

$\gtrless$

2) Test Statistic

$\hat{p} = 11/200 = 5.5\%$

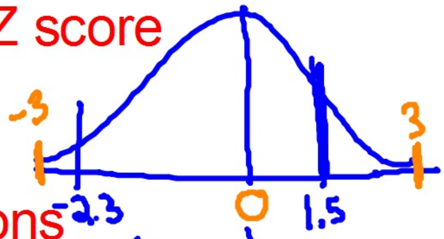
What is this test statistic used for?

Comparing a sample to our claim

$p = 20\%$

$\hat{p}$ p

What type of variable is this test statistic? Z score



Refresher: A z-score tells us...

how many standard deviations
a value is above/below its mean (claim)

Z = 1.5 Z = --2.3 $z = 4.3$

What types of Z-scores are "normal?" Between -3 and +3

Formula:

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

Example: $\hat{p} = 11/200 = 5.5\%$

$\cancel{p = 20\%}$

$\hat{p} = 18\%$
$p = 20\%$

$$Z = \frac{0.055 - 0.20}{\sqrt{\frac{(0.20)(0.80)}{200}}} =$$

$$Z = \frac{-0.145}{0.02828} = -5.1273$$

So Z-scores that are __small__ mean that the sample $\hat{p}$ is close to the parameter p, so we __believe__ the claim.

close to 0

So Z-scores that are __large__ mean that the sample $\hat{p}$ is far from the parameter p, so we __reject__ the claim.

far from 0

3) P-Value

Definition:

The probability of getting our sample (or something more extreme) if the claim is true

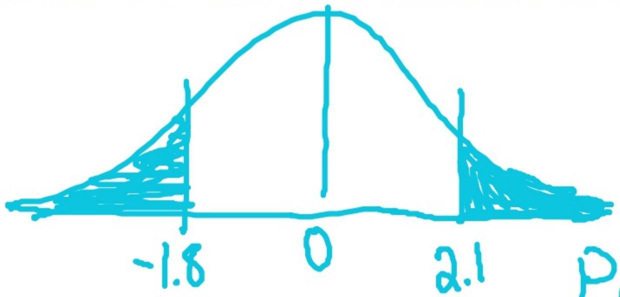
$p = 20\%$ small prob.

$\hat{p} = 18\%$ likely, large prob. $\hat{p} = 5.5\%$ unlikely

The smaller the p-value...

the more likely the claim is false

How do we use the test statistic to find the P-Value? (z)



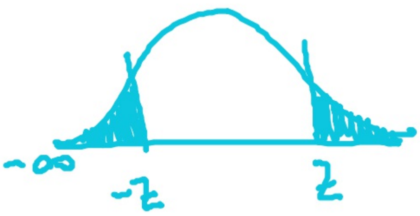
$P(z > 2.1) =$ p-value

$P(z < -1.8)$

What do we do on the calculator?

$P(Z \gtrless \underline{\text{test statistic}}) = \text{normalcdf}(LB, UB, 0, 1)$

Example:



$P(Z < -5.1273)$

$= \text{normcdf}(-E99, -5.1273, 0, 1)$

$= 1.47 \times 10^{-7}$

$= 0.000000147$

4) Conclusion:

What is a significance level?

- A number that we compare... the P-value to.

- We use this to help us decide...

  If the claim is true/false

What symbol do we use to denote this level?

ALPHA: α α α

The common significance levels are: $\alpha =$

0.01 0.05 0.10

If a significance level is not given, what level do we use?

Use 0.05

SO how do we decide if the claim is true?

* If the p-value is LESS THAN ALPHA, our claim is...

FALSE

* If the p-value is GREATER THAN ALPHA, our claim is...

ACCEPTED

Conclusion:

The two conclusions....

* If the P-Value is LESS THAN ALPHA: (REJECT Ho)

- We reject Ho b/c p-value of _________ is < α = _______.

- We have sufficient evidence that the true % of ________ is $>, <, \neq$ to __(claim)___%.

~~Ho: p = 20%~~

Ha: $p < 20\%$

p-value: 1.47×10^{-7}

$\alpha = 0.05$

We reject Ho b/c p-value of $1.47 \times 10^{-7} < \alpha = 0.05$.

We have suff. evid. that the true % of orange M+M's is less than 20%.

Conclusion:

The two conclusions....

* If the P-Value is GREATER THAN ALPHA:

~~believe~~

- We fail to reject Ho b/c p-value of _______ is > α = _____.

~~accept~~

- We do not have sufficient evidence that the true % of ________ is _____________ to __(claim)___%.

Ho: $p = 20\%$

Ha: $p < 20\%$

$\hat{p} = 15\%$

p-value: 0.20

Example 1: The National Board of Statistics claims that 15% of college students are classified as binge drinkers. They took an SRS of 17096 college students and found 3314 were classified as binge drinkers. Is there evidence that the percent has increased? Use $\alpha = 0.05$.

$p = 0.15$

$\hat{p} = \frac{3314}{17096} = 0.194$

$\alpha = 0.05$

$n = 17096$

Hypotheses:

$H_0: p = 0.15$

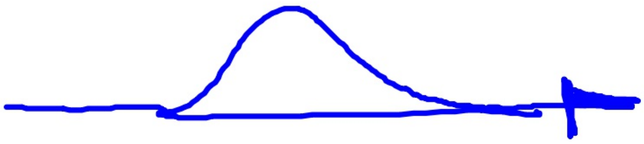
$H_a: p > 0.15$

Test Statistic:

$$z = \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.194-0.15}{\sqrt{\frac{(0.15)(0.85)}{17096}}} = 16.1172$$

P-Value:

symbol in Ha



$$P(z > 16.1172) = 1.02\times 10^{-58}$$

$\alpha = 0.05$

Conclusion:

We reject Ho b/c the p-value of $1.02 \times 10^{-58} < \alpha = 0.05$.

We have sufficient evidence that the true % of college Students who are binge drinkers is greater than 15%

~~Ho: p = 0.15~~

$H_a: p > 0.15$

Example 2: It has been claimed that 40% of all shoppers can identify a highly advertised trademark. If 10 of 36 shoppers were able to identify the trademark, is there evidence that the true proportion is now lower? Use the 1% level of significance.

$p = 40\% = 0.40 \quad n = 36$

$\hat{p} = 10/36 = 0.278 \quad \alpha = 0.01$

Hypotheses:

$H_0: p = 0.40$

$H_a: p < 0.40$

Test Statistic:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.278 - 0.40}{\sqrt{\frac{(0.40)(0.60)}{36}}} = -1.4942$$

P-Value:

$$P(z < -1.4942) = 0.0676$$

$$\alpha = 0.01$$

Conclusion:

We fail to reject Ho b/c the p-value of 0.0676 > α=0.01.

There is <u>not</u> enough evidence that the percent of people who can identify a trademark is less than 40%.

$H_0: p = 0.40$
$H_a: p < 0.40$

wrong

reject
p-val < α

$\underset{\text{p-val}}{0.0676} > \underset{\alpha}{0.01}$

Example 3: The proportion of people who are afraid of flying is claimed by the FAA to only be 35%. You do not believe this, and take a sample of 145 random adults and find that 70 of them are afraid of flying. Perform a statistical test of significance on the claim at the 0.05 significance level. Use α = 0.05.

$p = 0.35$

$\hat{p} = 70/145 = 0.483$

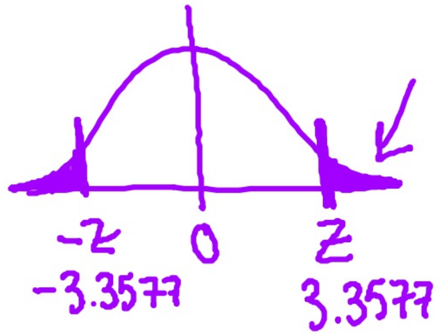
$n = 145$

$\alpha = 0.05$

$H_0: p = 0.35$

$H_a: p \neq 0.35$

$$z = \frac{0.483 - 0.35}{\sqrt{\frac{(0.35)(0.65)}{145}}} = 3.3577$$



$$2 \cdot P(z > 3.3577) =$$

$$2 \cdot \text{normalcdf}(3.3577, E99, 0, 1)$$

$$= 7.8605 \times 10^{-4}$$

$\underset{\text{p-val}}{0.00078605} < \underset{\alpha}{0.05}$

We reject Ho b/c the p-value of $7.8605 \times 10^{-4} < \alpha = 0.05$.

We have sufficient evidence that the true % of people who are afraid of flying is not equal to 35%.

~~$H_0: p = 0.35$~~

$H_a: p \neq 0.35$

WORKSHEET:

1. If 8 out of 40 patients suffered serious side effects from a new medication, is there sufficient evidence at the 1% level of significance to claim that the true proportion is not equal to 0.40? HW

2. A doctor claims that only 10% of all persons exposed to a certain amount of radiation will feel any ill effects. If 9 of 57 persons exposed to such radiation felt ill effects, is there sufficient evidence at the 0.05 level of significance that the true percent has increased?

3. A union spokesperson claims that 75% of union members will support a strike if their basic demands are not met. A company negotiator believes that the actual proportion of union members that will support the strike is less. He surveys 125 union members and finds that 85 support the strike. Test the claim at the 3% level of significance.